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Introduction

The "duct design" methods describe in *Industrial Ventilation* (IVM) [(1)] are designed to aid practitioners in selecting appropriate duct "sizes" (i.e., duct cross-sectional areas) and in selecting a fan for the system. With the exception of so-called "branch entry coefficients," the loss coefficients used in IVM[(1)] are based on laboratory studies of individual components of the system (e.g., elbows, straight ducts, hood entries, etc.). The branch entry coefficients are based on Alden's[(2)] estimates of plausible values.

The duct design procedure in IVM[(1)] is an attempt to model the behavior of the system created when the individual components are connected to each other. The "total pressure method" presented in the American Society of Heating, Refrigerating, and Air conditioning Engineers (ASHRAE's) *Fundamentals*[(3)] differs from the IVM[(1)] method only in its model of junction fittings. Other texts and manuals generally present the IVM[(1)] method without significant deviations.[(4)] Likewise, publications describing ventilation design spreadsheets,[(5)] and computer code[(6-9)] simply computerize the IVM[(1)] or ASHRAE[(3)] methods. The method of Tsal[(10)] is an algebraically manipulated version of the ASHRAE[(3)] total pressure method.

The authors found only one published study that compared predicted system performance to observed system performance. Koshland and Yost[(5)] found for an unspecified number of "different hood and duct sections" that the average error of prediction was 4% with maximum errors of ±20%. They did not describe the components or airflows used in these tests, making it difficult to generalize to other conditions.

The present work does not address the lack of field validation and it does not suggest fundamental changes to the IVM[(1)] methods. Instead, we assume that the IVM[(1)] methods are conceptually correct and reasonably accurate but could be modestly improved by incorporating the results of published studies on system modeling and by including the interactive models that will be discussed in succeeding sections. The former would have modest effects on most systems design to control particulates but would be moderately important for some plenum systems. The interactive modeling methods are modestly to moderately important when sizing ducts and selecting a fan but could be very useful when trying to understand the effects of modifying an installed system and for didactic demonstrations of system interactions with the fan and within branches.

FIGURE 1. Example duct system (ID numbers circled)

A Brief Review of IVM Static Pressure Calculations

This section briefly reviews the static pressure methodology presented in the IVM.[(1)] The total pressure method shown in ASHRAE *Fundamentals*[(3)] would compute almost the same values if the same velocity pressure coefficients were employed. Neither text accounts for interactions from downstream to upstream or for interactions with the fan. Given the similarities and that IVM methods are generally the basis for exhaust ventilation design, the rest of this work will focus on IVM methods.

The IVM strategy is to compute the pressure required at the junction fitting (SP_t) for each upstream pathway to the junction fitting based on the target airflow for that branch (Q_t), the density factor of the air (DF), and the loss coefficients for the branch. The target velocity pressure (VP_t) is computed first and used to compute SP_t. For example, for any branch, i, converging into a junction fitting, the static pressure required to achieve a given target airflow (Q_t) can be expressed as:

$$SP_{t_i} = -VP_{t_i}\,[1 + F_{h_i} + N_{el_i}F_{el_i} + F_{f_i}L_i + F_{en_i} + F_{misc_i}] - F_{slot_i}VP_{slot_i} - SP_{hood\ filter} \quad (1)$$

Where:

SP_{ti} = SP for path i

VP_t = velocity pressure computed from Q_t

F_h = velocity pressure coefficient for the hood to duct entry

N_{el} = number of equivalent 90 degree elbows

F_{el} = velocity pressure coefficient for one 90 degree elbow

F_f = velocity pressure coefficient for friction per unit length of duct

L_i = total length of the duct from hood connection to the junction fitting

F_{en} = velocity pressure coefficient for entry into the junction fitting

F_{misc} = velocity pressure coefficient for any other component in the branch duct

F_{slot} = velocity pressure coefficient for the sudden expansion from a narrow slot into a hood plenum

VP_{slot} = velocity pressure coefficient for the flow through the slot

$SP_{hood\ filter}$ = positive value of the differential pressure across filters in the hood (e.g., paint overspray filters)

For the system shown in Figure 1, one could compute the value of SP_t required to achieve each branch target airflow (Q_t), producing the values target branch pressures of SP_{t_1}, SP_{t_2}, SP_{t_3}, SP_{t_4}, SP_{t_5}, and SP_{t_6}. For example, the values for Branches 1 and 2 would be computed as (see Table I):

TABLE I. Industrial Ventilation Manual values

SPt_1 = -(0.86 in.w.g.)[1+0.25+(1.67)(0.19)+(0.0585/ft)(19 ft)+0.18 + 0]+0 = -2.47 in w.g.

SPt_2 =-(0.84 in.w.g.)[1+0.25+(1.67)(0.19)+(0.0446/ft)(19 ft)+0.18 + 0]+0 = -2.38 in w.g.

(-0.615 kPa and -0.593 kPa, respectively)

Even though one cannot directly measure the so-called "governing pressure" at which converging flows reach a common pressure, it has been demonstrated with empirical data[11,12] that the system does, indeed, act as if there were a common, single pressure, SP_J at a junction fitting where two air flows converge. As a corollary, the higher the value of

SP_J, the higher the flow through any duct connected to that junction. The most prudent value to select for SP_J is the minimum of the target values for the paths leading up to it. For example, for Paths k and i converging at a common pressure $SP_{J_{k,i}}$:

$$SP_{J_{k,i}} = \text{minimum of } \{SP_{t_k}, SP_{t_i}\} \quad (2)$$

Note that since both SP_t values must be negative, the minimum value has the greater magnitude. For example, for Branches 1 and 2 in Figure 1:

$$SP_{J_{1,2}} = \text{ minimum of } \{-2.47 \text{ in w.g., } -2.38 \text{ in w.g.}\} = -2.47 \text{ in w.g.} \quad (-0.615 \text{ kPa})$$

Unless all pathways have exactly the same pressure requirements, the pathways with lower magnitudes of SP_t will have more pressure available than is required to obtain their target airflows. As result, more air will flow than was desired. One can estimate the "corrected" flow through each pathway as:

$$Q_{corr_i} = Q_{t_i} (SP_{J_{i,j}} / SP_{t_i})^{0.5} \quad (3)$$

Where i can be any of the converging ducts, whether branch or submain

For example, Branches 1 and 2 flow into submain 10 (see Figure 1) and:

$$Q_{corr_2} = Q_{t_2} (SP_{J_{1,2}} / SP_{t_2})^{0.5} = 500 \text{ cfm } (-2.47 \text{ in w.g.}/-2.38 \text{ in w.g.})^{0.5} = 509 \text{ cfm } (14.4 \text{ m}^3/\text{min})$$

For each path the corrected airflow will either equal or exceed the target airflow, depending on whether the magnitude of the junction pressure (SP_J) equals or exceeds the magnitude of SP_t.

The airflow (Q_{t_n}) in the downstream submain, n, is the sum of the "corrected" branch airflows. For example, if ducts i and k are upstream of Submain n:

$$Q_{t_n} = Q_{corr_i} + Q_{corr_k} \quad (4)$$

Note that the value of Q_t for a submain is NOT the sum of the upstream Q_t values. Next, Q_{t_n}, the density and cross-sectional airflow in the submain are used to compute the pressure requirement for Path n up to the next junction (e.g., $SP_{J_{o,d,e}}$):

$$SP_{t_n} = SP_{J_{k,i}} - VP_t \left[N_{el_n} F_{el_n} + F_{f_n} + F_{en_n} + F_{misc_n}\right] - VP_n + (Q_{corr_i} VP_{corr_i} + Q_{corr_k} VP_{corr_k}) / Q_{t_n} \quad (5)$$

Where: VP_{corr} = velocity computed based on corrected Q value

Values of Q_{corr} for the next downstream converging junction are computed using Equation 3. For example, if the converging ducts are branch 3 and submain 10 from Figure 1, then:

$$Q_{corr_3} = Q_{t_3} (SP_{J_{10,3}} / SP_{t_3})^{0.5}$$

$$Q_{corr_{10}} = Q_{t_{10}} (SP_{J_{10,3}} / SP_{t_{10}})^{0.5}$$

This approach will determine the required or "target" fan airflow ($Q_{t_{fan}}$) and fan total pressure ($TP_{t_{fan}}$) within the accuracy limitations of the loss coefficients used in the equations. However, the predicted airflows for some branch ducts may be erroneous for reasons discussed in following sections. In addition, the method has inaccuracies and unnecessary limitations in modeling airflow behavior at junction fittings. The following sections discuss those errors and limitations and suggest more accurate methods.

Table I is a solution of the first three branches of the system in Figure 1 using current IVM methods. Table I does not include the solution for all of the system since Branches 4 and 5 end in a double-lateral junction (see later sections), which are not allowed by IVM.

Improvements to the IVM Method (Without Interactions)

Although the IVM method apparently does an adequate job of assisting in selecting duct sizes and providing pressure and flow information needed to select the fan, it has several limitations and inaccuracies in addition to the lack of interactive modeling to be discussed in a later, separate section.

The modest improvements described in this section do not have profound effects on results, but they should make the predictions somewhat more accurate, especially for plenum systems. The gain in accuracy varies in complex ways with conditions but in the authors' experience the difference in predictions is typically is 0.5%-5%.

Q-Correction Exponent

IVM[1], ASHRAE *Fundamentals*[3] and other texts employ an exponent of 0.5 for the "Q correction" equations (Equation 3). This exponent is based on the assumption that pressures are proportional to airflow squared. Since the branch and other duct pressures are due to a combination of both friction and dynamic losses, an exponent of 0.513 gives a better estimation.[13] The improvement in accuracy is modest but requires no additional effort when programming software or in setting up electronic spreadsheets.

Using the exponent of 0.513, Equation 3 becomes:

$$Q_{corr_i} = Q_{t_i}\,(SP_{J_{k,i}}/SP_{t_i})^{0.513} \quad (6)$$

Where i can be any of the ducts converging to the junction fitting, whether branch or submain

It will be helpful later in this exposition to define Q_{ratio_i} as:

$$Q_{ratio_i} = (SP_{J_{k,i}}/SP_{t_i})^{0.513} \quad (7)$$

Using Equation 7, Equation 3 can be restated as:

$$Q_{corr_i} = Q_{t_i} * Q_{ratio_i} \quad (8)$$

Double-Lateral Junctions

Single-lateral junctions connect two upstream ducts to a submain or main. Examples are the junction of ducts 1 and 2 and the junction of ducts 3 and 10 in Figure 1. However, in some systems it would be convenient to connect two laterals to the same junction fitting instead of using two single-lateral junctions. An example is the junction of ducts 4, 5, and 20 in Figure 1. Indeed, it is often necessary to install unnecessary lengths of duct and an additional elbow to employ two single-lateral junctions when one double-lateral would have served. The fittings that connect three upstream ducts are called "double-lateral" or "bilateral" junctions.

FIGURE 2. Single and double-lateral junctions

Despite their potential convenience, practitioners generally avoid double-lateral junctions because IVM labels them as a type of branch entry to "avoid." However, Guffey and Curran[14] demonstrated with experimental evidence that there was no energy advantage to using two single-lateral junctions instead of one double-lateral junction. Indeed, in cases where the layout of the system makes double-lateral junctions convenient, the energy costs of two single-lateral junctions would be substantially greater than one double-lateral junction because extra elbows and duct lengths are required to accommodate two single-lateral junctions. Furthermore, the IVM static pressure calculation scheme works quite well for double-lateral junctions if empirically-based velocity pressure coefficients are employed.[14]

FIGURE 3. Avoiding a double-lateral junction

If double-lateral junctions are to be allowed, the IVM procedure must be modified to accommodate them. First, it is necessary to re-state Equation 2 for the more general case of three converging ducts (e.g., ducts k, i, and j):

$$SP_{J_{a,b,c}} = \text{minimum of } \{SP_{t_a}, SP_{t_b}, SP_{t_c}\} \quad (9)$$

Where:
- a = identifies a duct converging into the junction fitting
- b = identifies a duct converging into the junction fitting (omit for single-lateral junctions)
- c = identifies a collinear duct converging into the junction fitting

For example, the third junction in Figure 1 is identified as $SP_{J_{20,4,5}}$. As is shown in Table II, the computed value of $SP_{t_{20}}$ = -3.65 in w.g., SP_{t_4} = -1.99 in w.g., and SP_{t_5} = -2.45 in w.g. Using Equation 9:

$$SP_{J_{20,4,5}} = \text{minimum } \{-3.65 \text{ in w.g.}, -1.99 \text{ in w.g.}, -2.45 \text{ in w.g.}\} = -3.65 \text{ in w.g.}$$

(-0.909 kPa)

TABLE II. Improved Industrial Ventilation Manual values

Second, the IVM procedure for "Q-correction" computations for double-lateral junctions should be modified as follows:

$$Q_{corr_a} = Q_{t_a}\,(SP_{J_{a,b,c}}/SP_{t_a})^{0.513} \quad (10)$$

Where a can be any of the 2 or 3 converging ducts whether branch or submain

For example, for Branch 5:

$$Q_{corr_5} = Q_{t_5}\,(SP_{J_{20,4,5}}/SP_{t_5})^{0.513}$$

Substituting the values for the unknowns in Equation 10 using the values in Table II produces:

$$Q_{corr_5} = 500\text{ cfm }(-3.65\text{ in w.g.}/-2.45\text{ in w.g.})^{0.513} = 613\text{ cfm } (17.3\text{ m}^3/\text{min})$$

As before, it will be useful later to expand the definition of Q_{ratio} to include double-lateral junctions:

$$Q_{ratio_a} = (SP_{J_{a,b,c}}/SP_{t_a})^{0.513} \quad (11)$$

Third, the airflow in the downstream duct is now the sum of three airflows, not two, so Equation 4 is re-stated as:

$$Q_{t_n} = Q_{corr_a} + Q_{corr_b} + Q_{corr_c} \quad (12)$$

FIGURE 4. Pressure changes at a junction fitting

Junction Losses and Coefficients

There are two "losses" at junction fittings due to the entry from the lateral, the difference in pressure upstream of the junction fitting (SP_{en}) and the losses downstream of the junction fitting (SP_k). SP_{en} does not actually represent energy losses. Instead, it expresses the small differences in static pressures that occur upstream of the same junction.[12] It can be computed from:

$$SP_{en} = F_{en} \times VP \quad (13)$$

Where: SP_{en} = pressure change due to entering junction fitting

VP = velocity pressure in the lateral duct

F_{en} = empirically-determined velocity pressure coefficient

The F_{en} values in IVM were originally developed as a "temporary expedient" by Alden[2] in hopes that a better model based on physics and empirical coefficients would soon

replace it. Although he stated that they were "admittedly incomplete and unsupported by sound experimental evidence," F_{en} values based on them are still used in IVM at this writing. Alden used F_{en} values to compute losses in the submain downstream of the junction fitting, but IVM uses them to compute losses in the lateral duct upstream of the junction fitting. A review of IVM archives failed to show the genesis of this deviation from Alden's recommendation.

Guffey and Fraser(15,16) analyzed an extensive set of empirical data that included 75 junction fittings that had been tested at dozens of flow conditions apiece. Their analyses showed that the current IVM F_{en} values have a nearly zero correlation to empirical values. The empirical values of F_{en} proposed by Guffey(12) had an adjusted R-squared of over 98% for the same data. They varied with ratios of areas at the junction as well as the lateral entry value, which is consistent with values published by ASHRAE.(3) The Alden/IVM values vary only with entry angle.

The second problem with the current IVM junction method is that it fails to include a term for the loss downstream of the junction due to the junction (SP_k), thereby unintentionally predicting zero losses for junctions (an impossibility). Guffey and Fraser(16) proposed a model for downstream losses that fit the same data set discussed above (this time for downstream losses) with an R-squared of over 98%. That model for junction losses can be applied to fittings with any number of upstream ducts. Applied to fittings with three upstream ducts, that model is:

$$SP_k = (K_i Q_{corr_i} VP_{corr_i} + K_j Q_{corr_j} VP_{corr_j} + K_k Q_{corr_k} VP_{corr_k})/Q_{t_n} \qquad (14)$$

Where: K_k, K_j, K_i = junction pressure coefficients(12)

VP_{corr} = velocity computed based on the corrected Q value

Computing Friction Losses

The IVM's preferred method of computing friction losses uses Loeffler's equation and coefficients,(17) which is a reasonably good approximation for ducts with a roughness of 0.0005 ft (0.00015 m). Guffey(18) provided a somewhat different equation with coefficients for many different duct roughnesses. Since the latter used two or more reference velocities to divide the range of values, the degree of extrapolation is much reduced, producing lower errors of approximation (< 4%) than Loeffler when applied to relatively low (e.g., 2000 ft/min) (609.6 m/min) or relatively high duct velocities (e.g., 5000 ft/min) (1524 m/min). Either approach is satisfactory for hand-calculator computations but both require input of at least two coefficients for each roughness one might employ. Haaland's approximation(19) is much more complicated but is also is more accurate over a broad range. More importantly, it computes values for different roughnesses without substituting different coefficients and exponents.

Haaland's equation can be modified as follows for use as a friction loss coefficient, F_f:

$$Ff = \frac{\left\{-1.8\log_{10}\left[\frac{6.9}{\text{Re}}+\left(\frac{Roughness}{3.7D}\right)^{1.11}\right]\right\}^{-2}}{D} \quad \text{.......} (15)$$

Where: Re = Reynolds number

Roughness = equivalent smoothness of duct material

D = duct hydraulic diameter

Note: roughness and diameter must have consistent units

The static pressure differential due to friction is then computed as:

$$SP_f = F_f \text{ x } VP \quad \text{.......} (16)$$

Equation 15 is complex, but it is necessary to enter it into a spreadsheet or code it into software only once. Once successfully done, it can be copied and placed into other spreadsheets or programming code.

Interactive Pressure Calculations

The procedures above account only for upstream to downstream effects. That is, the downstream airflows and pressures were determined solely by computations involving upstream converging paths. Two things are missing: 1) applying Q_{corr} from downstream to upstream, and, 2) modifying airflows and pressures to account for interactions between system resistance and fan performance.

Applying Q_{corr} from Downstream to Upstream

If the magnitude of the pressure requirement (SP_t) for a submain is less than magnitudes of the pressure requirements of any of the other ducts converging at the same junction, then according to Equations 6 and 10, the airflow through the submain will rise above its target levels. Since the airflow through the submain all comes from upstream branch ducts, an increase in airflow for a submain requires a proportional increase in airflow for every duct upstream of that submain. In fact, the proportion would be the value of Q_{ratio_i} for the submain (see Equation 11).

If more than one downstream submain had $Q_{corr} > Q_t$, then the airflows of branches upstream of them would be multiplied by two or more submain Q_{ratio} values. Suppose, for example, that downstream of junction a,b,c there are many more junctions in sequence and that from upstream to downstream we encounter "*w*" values of Q_{ratio_i} from Q_{ratio_1} to Q_{ratio_w}. The value of any Q_{corr_i} thus would be:

$$Q_{corr_i} = Q_{t_i} \text{ x } Q_{ratio_i} \text{ x } Q_{ratio_{dn(1)}} \text{ x } Q_{ratio_{dn(2)}} \text{ x } \ldots \text{ x } Q_{ratio_{dn(w)}} \quad \text{.......} (17a)$$

Where: dn(1) = duct immediately downstream from duct i

dn(2) = duct immediately downstream from duct dn(1)

dn(w) = duct immediately downstream from duct dn(w-1)

For example, for Branch 2 in Figure 1 the downstream submains are 10, 20, 30, 40, 50, 60, 65, and 70. Since there are no junctions at the ends of ducts 40, 50, 60, 65, and 70, Q_{ratio} for each would be unity. The values of Q_{ratio} for submains 10, 20, and 30 have the possibility of exceeding unity. Substituting into Equation 17a:

$$Q_{corr_2} = Q_{t_2} \times Q_{ratio_2} \times Q_{ratio_{10}} \times Q_{ratio_{20}} \times Q_{ratio_{30}} \quad (17b)$$

Note that Q_{corr_i} now can be computed only by iterative solution since each Q_{ratio} value is affected by Q_{corr_i}.

As before, Equation 17b can be simplified by defining another variable, Q_{mult}, as a recursive computation:

$$Q_{mult_i} = Q_{ratio_i} \times Q_{mult_{dn(1)}} \quad (18)$$

Substituting into Equation 17a produces:

$$Q_{corr_i} = Q_{t_i} \times Q_{mult_i} \quad (19)$$

Note that Equation 19 does recapitulate Equation 17b. For example, if Ducts 10, 20, and 30 are in sequence from upstream to downstream, then for Branch 1:

$$Q_{corr_1} = Q_{t_1} \times Q_{mult_1}$$

$$Q_{mult_1} = Q_{ratio_1} \times Q_{mult_{10}}$$

$$Q_{mult_{10}} = Q_{ratio_{10}} \times Q_{mult_{20}}$$

$$Q_{mult_{20}} = Q_{ratio_{20}} \times Q_{mult_{30}}$$

$$Q_{mult_{30}} = Q_{ratio_{30}} \times 1$$

Hence, for example, substituting in the definitions of Q_{mult} for the downstream submains, the flow through Branch 1 would be computed from:

$$Q_{corr_1} = Q_{t_1} \times Q_{ratio_1} \times Q_{ratio_{10}} \times Q_{ratio_{20}} \times Q_{ratio_{30}} \times 1$$

Equation 19 can be computed without difficulty within a properly written ventilation design software program, but using it within current spreadsheet programs is more of a challenge. Equation 19 is much too difficult to implement directly in current spreadsheet programs since the results of upstream pressure computations are affected by downstream results and vice-versa. Spreadsheet programs such as Microsoft Excel© have difficulties with iterative solutions that involve a large number of calculations. However, a "work-around" for spreadsheet models is shown in a succeeding section.

Effects of Fan Performance on System Flows

So far, interactions about duct resistances have been considered but not interactions with the fan. The effect of changes in fan performance can be estimated by considering that the airflows throughout a duct system maintain the same fractions of the fan airflow.[11]

For example, if there is no change in resistance the airflow through the fan and through Branch 1 at fan rotation rates, ω_{init} and ω_{final} would be:

$$Q_{fan_{final}} = Q_{fan_{init}} (\omega_{final}/\omega_{init}) \quad (20)$$

$$Q_{1\_final} = Q_{1\_init} (Q_{fan_{final}}/Q_{fan_{init}}) \quad (21)$$

This relationship produces less than 5% errors even when fan airflow is doubled unless there are large non-quadratic losses (e.g., hood filters). Since it is unusual to deal with multiple branch systems with substantial hood filter losses, we can almost always ignore that caveat.

Applying this last correction, Equation 19 becomes:

$$Q_{final_i} = Q_{t_i} \text{ x } Q_{mult_i} \text{ x } (Q_{fan_{final}}/Q_{fan_{init}}) \quad (22)$$

Where: Q_{final_i} = the expected flow through Duct i for the specified duct system, fan, and fan rotation rate

The static pressure for any pathway, i, can now be estimated from an algebraic manipulation of Equation 11:

$$SP_{corr_i} = SP_{t_i} \text{ x } (Q_{final_i} / Q_{t_i})^{1.95} \quad (23)$$

Equations 18 and 19 account for interactions within the duct system and the effects of changes to fan output. The remaining step is to model the effect of the duct system on fan performance, allowing one to compute Q_{final}.

Computing Measurable Values

The IVM procedure computes the static pressure at the hood (SP_h) and the junction (SP_J). Neither computation is a reliable prediction of values one could expect to measure in the installed system. Both are computed as idealized values using Equation 1 for SP_J and the following for SP_h:

$$SP_{h_i} = -VP_{t_i} [1+ F_{h_i}] - F_{slot_i} VP_{slot_i} - SP_{hood\ filter} \quad (24a)$$

The computation for SP_h is not always a good basis for prediction because: 1) it ignores the effect of the Q_{corr} computations, 2) it ignores the fact that reliable measurements cannot be made at the point where the hood connects to the duct. As is pointed out in IVM, SP_h should measured 2-4 duct diameter's (2D-4D) from the connection. In addition, sometimes there is an elbow immediately downstream of a hood connection, forcing the selection of measurement points downstream of the elbow.

Taking all those considerations into account produces the following:

$$SP_{h_{final}} = -(Q_{final} / Q_t)^{1.95} \ \{ VP (1 + F_h + L_h F_f + N_{el} F_{el}) + F_{slot} VP_{slot} \} - SP_{hood\ filter} \quad (24b)$$

Where: $SP_{h_{final}}$ = the expected hood static pressure in the installed system

L_h = distance between the SPh measurement location and the duct-to-hood connection

N_{el} = number of equivalent 90 degree elbows between the SP_h measurement location and the duct-to-hood connection

Computing a measurable value near the junction fitting has similar problems. Equation 1 can be modified to take into account that measurements must be taken some distance upstream of the junction, but doing the same for submains would take many, many steps. Instead, it is more convenient to use the results of computing SP_J and correct it to find the pressure at the end of the duct (SP_{end}) before the junction fitting (note that SP_J is a negative value and that $SP_{hood\ filter}$ is a positive value):

$$SP_{end_{final}} = (Q_{final} / Q_t)^{1.95} \ \{SP_J + SP_{hood\ filter} + VP(\ L_h\ F_f + N_{el}\ F_{el})\} - SP_{hood\ filter} \quad (25)$$

Where: $SP_{end_{final}}$ = the expected static pressure just upstream of the end of the duct just before the junction, air-cleaner, or fan

L_h = distance between the SPend measurement location and the terminating point of the duct

N_{el} = number of equivalent 90 degree elbows between the SP_{end} measurement location and the terminating point of the duct

Modeling Fan Performance

In ventilation texts and manuals,[1,3,4] it is commonly stated that a fan operated at a given rotation rate performs at a level determined by the intersection of the "system curve" with the "fan curve." The system curve is plotted as the computed fan static pressure (SP_{fan}) or fan total pressure (TP_{fan}) at the fan versus the airflow entering the fan. Usually, it is assumed that the system curve can be computed from an extrapolation of $SP_{fan} \propto Q^2$. A contrived example plot is shown as Figure 5.

FIGURE 5. Fan curve intersection with extrapolated system curves

However, no real duct system has pressures that are truly proportional to Q^2. As has been shown elsewhere,[13,18] friction losses account for a substantial fraction of total system pressure, and friction losses are roughly proportional to about $Q^{1.9}$ for ducts used in most exhaust ventilation systems. More importantly, some systems have baghouses, whose static pressure drop varies over some range of pressures during a cleaning cycle. For a baghouse, the pressure drop across the filters is roughly linear with airflow. For such systems, the system curve is decidedly non-quadratic.

The "fan law" that states that airflow changes proportionately with fan rotation rate[20] predicts the effects of changing fan speed only if all losses are proportional to Q^2. Since real duct system losses are not truly proportional to Q^2, the fan laws are not precisely predictive for real systems, particularly if a baghouse is connected to the duct system.

Furthermore, even if system losses were truly proportional to Q^2, it would be difficult to use the fan pressure versus system pressure curves to predict the effects of changes to system resistance to fan airflow. For that, it is better to relate Q_{fan} to a measure of system resistance, X_{sys}:[21]

$$X_{sys} = TP_{fan}/VP_{inlet} \qquad (26)$$

Where: X_{sys} = equivalent loss coefficient for the duct system

TP_{fan} = total pressure for the fan

VP_{inlet} = velocity pressure at the fan inlet

Using the values published by fan vendors in their tables of performance, it is possible to plot Q_{fan}, TP_{fan}, and fan efficiency with changes in X_{sys}. Since a given fan model can be set up for many different rotation rates (ω) and for different wheel diameters (D_w), it is helpful to normalize for rotation rate and D_w. According to the well-known "fan-laws,"[20] Q_{fan} is proportional to ω and to D_w^3, and TP_{fan} is proportional to ω^2 and to D_w^3. Hence, fan airflows can be normalized for rotation rate and wheel diameter with:

$$Q_{norm} = (\text{Table value of } Q_{fan})/\ \omega/\ D_w^3 \qquad (27a)$$

Where: D_w = diameter of the fan wheel

Note that it is assumed that manufacturers increase the wheel depth with D_w. If the wheel depth remains constant as D_w is changed, Equation 27a would become:

$$Q_{norm} = (\text{Table value of } Q_{fan})/\ \omega/\ D_w^2 \qquad (27b)$$

Fan pressures can be normalized for rotation rate and wheel diameter with:

$$TP_{norm} = (\text{Table value of } TP_{fan})/\ \omega^2/\ D_w^2 \qquad (28a)$$

$$SP_{norm} = (\text{Table value of } SP_{fan})/\ \omega^2/\ D_w^2 \qquad (28b)$$

The graphs of Q_{norm} and TP_{norm} for a particular fan (name not revealed) are depicted as Figure 6. Note that airflow declines and TP_{fan} increases with increases in system resistance, as expected. In Figure 6, note that the fan's efficiency peaks at a particular value of X_{sys}, suggesting that X_{sys} could provide a reasonable basis for selecting among competing fans.

FIGURE 6. Normalized performance curves with system resistance

Note that the values plotted in Figure 6 are plotted for dozens of different rotation rates yet form continuous curves, illustrating that normalizing with ω removes it as a variable for Q_{norm} and TP_{norm}. To the degree that the fan laws are correct, fan efficiency is independent of rotation rate. There are complications in employing these normalized values for different wheel diameters because the diameter of the fan inlet also changes with D_w, affecting the TP_{fan} computation. Since the value of X_{sys} computed from the duct

system is based on a specific fan inlet duct size, it would be most accurate to recompute TP_{fan} for the system with matching fan inlet duct sizes if one wishes to investigate the suitability of a particular fan for a given system. However, that consideration is irrelevant here since the goal is to model a system connected to a specific fan, for which D_w would not change.

The next step is to develop a mathematical relationship between Q_{norm} and X_{sys}. As shown in Figure 7, Q_{norm} is highly linear with $\log(X_{sys})$. Regression analysis for this particular fan shows an R-square of over 99%. The authors have found similar linearity for other vendors and other models of radial and backwardly inclined centrifugal fans over their useful ranges of application but have not made a systematic investigation to determine for which vendors and models the linearity is adequate and for which it is not.

FIGURE 7. Normalized fan performance and the log of system resistance

For the fan shown in Figure 7, the regression coefficients and model are:

$$Q_{fan} = [a - b \log(X_{sys})]\, \omega^2 D_w^3 \qquad (29)$$

Where: a = regression intercept

b = regression slope

Values from Equations 22, 23, 24b, 25 and 29 can be used to model all interactions within the system as well as the interactions between the system and the fan.

Revised Calculation Procedure

Putting these improvements together, the calculation scheme outlined earlier remains mostly familiar in appearance but has some important differences.

Modeling Interactions with Spreadsheet Programs

Equations 22, 23, 24b, and 27 cannot be computed directly with current spreadsheet computer programs without extensive user programming (e.g., using Microsoft's© Visual Basic for Applications). This is true because $Q_{fan_{final}}$ depends on Q_{final} for each duct and vice-versa. Hence, only iterative solutions are possible for direct application of the equations. Current spreadsheet programs have a very limited ability to do iterative ("circular") computations and will quickly bog down for this particular application. However, as will be demonstrated, it is possible to create a non-iterative approach that can work within a useful range of conditions. The key is to create additional variables (e.g., columns of data in the spreadsheet) for spreadsheets. Instead of computing a value of Q_{corr} using iterative solutions, a series of equations are solved sequentially so that the results of equations are not needed for their own solution. For this, one should do the following:

1. Set up a spreadsheet in the typical fashion (see Table II) to compute pressures and flows all the way down to the fan.
2. In doing so, compute duct velocity (V) from Q_{t_i} and the cross-sectional area (A) of each duct.
3. Compute the velocity pressure in each duct using V and compute SP_t using Equation 1 or Equation 5.
4. In another column, compute SP_J for each junction using Equations 2 or 9 and assign the value to each duct converging at that junction.
5. Compute Q_{corr} for each duct in its own column using Equation 3. In the adjacent column, compute Q_{ratio} for each duct using Equation 7.
6. After computing the duct pressures and flows, compute the "target" values, $TP_{t_{fan}}$, $SP_{t_{faninlet}}$, and $Q_{t_{fan}}$ as is typically done.
7. Compute X_{sys} using Equation 26 with $TP_{t_{fan}}$ and VP_t for the fan inlet duct.

   $$X_{sys} = TP_{fan}/VP_{inlet}$$

   $$X_{sys} = 11.72 \text{ in w.g.}/0.27 \text{ in w.g.} = 43.41$$

8. Using predetermined regression coefficients for a specific fan, predict system airflow (Q_{pred}) using X_{sys}, D_w, ω, and Equation 29.

   $$Q_{fan} = [a - b\log(X_{sys})]\,\omega^2 D_w^3$$

9. In another column, compute Q_{mult_i} for the complete pathway from each duct to the fan by multiplying together all of the values of each Q_{ratio_i} times the value of Q_{mult_i} computed for the next duct downstream ($Q_{mult_{dn(i)}}$):

   $$Q_{mult_i} = Q_{ratio_i} \text{ x } Q_{mult_{dn(i)}}$$

   Note that Q_{mult_i} is recursive.

10. In another column, compute Q_{final} for each duct from:

    $$Q_{final_i} = Q_{mult_i}\,(Q_{pred}/Q_{t_{fan}})$$

11. As a check, input $Q_{t_i} = Q_{final_i}$ for each branch duct and recompute Q_{corr_i} for each branch duct using the initial columns of the spreadsheet. There should be negligible difference now between the resulting values of Q_t, Q_{corr}, and Q_{final} for each branch. Likewise, the new fan airflow should be negligibly different from Q_{pred}.

The spreadsheet now should show the interactive effects of making any change to any duct and to the fan. For example, if the user doubles the length of a branch duct, the velocity and airflow through that duct should decrease and the velocity and airflows through all other branch ducts should increase.

The chief limitations to this approach are:

1. It can become cumbersome to set up. Indeed, it is probably not practical to model a large system this way.
2. The individual setting it up must have a good understanding of these methods.
3. X_{sys} can be substantially in error if a baghouse is part of the system and the airflow through it changes substantially because of a large change in fan rotation rate.

The latter difficulty arises only when the rotation rate of the fan is changed substantially. The error occurs because the steps outlined above use the value of X_{sys} as determined in columns whose values do not reflect the effects of a change in fan speed. If the air-cleaner loss is quadratic (exponent = 2), this introduces no error since the value of X_{sys} would be the same regardless of level of airflow. If the exponent is zero (constant pressure) or unity (filter loss), then the value of X_{sys} computed at the original fan rotation rate would be somewhat different from the value computed at the new rotation rate. Extensive modeling by the authors showed less than 1% error for a 40% change in fan speed even under the worst likely conditions (e.g., half of system pressure due to a constant filter loss). With a linear filter loss, the error is less than 3% when the fan speed is doubled. The error could be eliminated by manually inputting the new baghouse pressure, but it is unlikely that it would be necessary for realistic conditions. More importantly, the error does not affect the distribution of airflows to the branch ducts, only the total flow.

A following section illustrates the use of such a spreadsheet to model the effects of reducing a branch duct diameter and reducing the diameter of a submain duct.

Implementing Interactions in Computer Code

The authors are aware of only one computer program for ventilation design known that currently does these calculations,[22] so the methods used for it will be described here. The approach has much in common with the spreadsheet modeling described above. The important difference is in computing velocity. Step Number 2 is replaced with:

2. Compute duct velocity from Q_{final} and the cross-sectional area (A) of each duct:

$$V_i = (Q_{final_i} / A_i)$$

$$\text{Where: } Q_{final_i} = Q_{t_i} \text{ if } Q_{final_i} = 0$$

Note that Q_t becomes the seed value for Q_{final}. If one computes from Step 2 all the way through numerous times, the values of Q_{final_i} in Step 10 will asymptotically approach the true value of Q_{final} for each duct.

One advantage of this substitute Step 2 is that the actual value of the duct velocity is always used to compute F_f, giving more exact answers than are practicable with a spreadsheet. A more critical advantage is that the correct airflow is always used to compute pressure across the air-cleaner. As a result, the value of X_{sys} can be

automatically updated with complete accuracy. Both advantages together allow solutions that have no errors due to approximations or extrapolations. The only errors are those due to the prediction error of the fan regression equation, those due to use of published loss coefficients, and those due to the (unknown) limitations of the assumptions underlying static pressure calculations.

Demonstration Problems

This example application of the method employs the spreadsheet in Table III to illustrate the effect of effecting some common practices or occurrences without changing fan speed. The spreadsheet can also accommodate changes to fan speed.

Demonstration of the effects of reducing a branch duct diameter

Suppose that it is proposed to reduce the duct diameter for Branch 1 from 4 inches to 3.5 inches in attempt to end a severe plugging problem from a hygroscopic material. The assumption is that reducing a duct diameter always produces an increased duct velocity.

As shown in Table III, the branch duct velocity would increase by less than 2% while the Branch 1 airflow would fall by 22%. In this case, the result would be little or no improvement in plugging and a possibly substantial reduction in hood effectiveness. Note that the reduction in Branch 1 airflow guaranteed a reduced airflow in Submain 10, leading to a reduction in Submain 10 duct velocity of 4.4%. Reducing the duct size would produce disappointing results.

For a second example, suppose we need to increase the airflow through Branch 1 by 40% but do not wish to see the velocity fall substantially. A 40% shift could be obtained using dampers on the rest of the branches, but that would produce a large increase in fan pressure. We reluctantly conclude that a larger diameter duct must be used for Branch 1. However, we are concerned that the duct velocity will fall sharply as an unintended consequence of increasing the cross-sectional area. As shown in Table III, airflow in Branch 1 would increase by 49% and the velocity would fall by 4.8%, a modest change that may or may not be acceptable.

For a third example, consider the effects of reducing or increasing the diameter of Branch 6, the last branch before the fan. As shown in Table III, reducing its diameter produced higher velocities in all ducts except Submain 40. It also reduced the airflow by 22% in Branch 6. Note that increasing the diameter of Branch 6 had almost exactly converse effects. All other branch velocities fell by nearly 9%, except Submain 40, which increased slightly due to a small increase in overall airflow. The slight increase in overall airflow was due to the fan's response to the reduced system resistance.

TABLE III. Percentage changes in airflows and velocities with a change in a duct diameter

For the fourth and last example, consider the consequences of reducing the diameter of Submain 20 from 8 inches to 7 inches. Note that its velocity increased by 26%, while

upstream duct airflows and velocities fell modestly (4%) and downstream branch airflows and duct velocities increased modestly.

The lesson to be learned from these examples is that the effects of changing duct diameters are complex. In particular, reducing a duct diameter one incremental size typically will modestly increase a branch duct's velocity while dramatically reducing its airflow.

Conclusion

The methods described here for modeling system interactions and system/fan interactions can be employed in either spreadsheets or in programming code. The latter eliminates approximations and extrapolation errors. Although the spreadsheet approach would be difficult to use in modeling large systems, it can be extremely useful in modeling smaller systems to explore the effects of common strategies.

Four example problems demonstrated both the utility of the approach and the need to use interactive modeling when teaching ventilation design. Work in progress will demonstrate how such modeling can be used to compare common airflow balancing strategies.

The authors recommend that the ACGIH Committee on Industrial Ventilation and authors of other texts consider adding bilateral junctions as an integral part of their procedures and that they list the other improvements suggested here as "advanced" procedures for more sophisticated readers. Given the lack of basis for current IVM junction computations, the authors recommend that the procedures for junction calculations discussed here be substituted for current methods and coefficients.